RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

M.A./M.Sc. FOURTH SEMESTER EXAMINATION, MAY 2023

SECOND YEAR (BATCH 2021-23)

MATHEMATICS

Date : 19/05/2023

Time : 11.00 am – 1.00 pm

Paper : CC19

Full Marks : 50

[All the symbols have their usual meaning]

Answer **all** the questions, maximum one can score 50.

1. Let $f(x)=(\cos\pi x,\sin\pi x), g(x)=(\cos\pi x,2\sin\pi x)$ and $h(x)=(\cos\pi x,-\sin\pi x)$ be three paths in the punctured plane. Discuss about the path homotopy among them. [5]

2. If X is path connected and x_0, x_1 are two points of X, then $\pi_1(X,x_0)$ is isomorphic to $\pi_1(X,x_1)$- true or false? Discuss with acceptable reasons. [5]

3. Let $A\subseteq X$; suppose $r: X\to A$ is a continuous map such that $r(a)=a$ for each $a\in A$. If $a_0\in A$, show that $r_*:\pi_1(X,a_0)\to\pi_1(A,a_0)$ is surjective. [5]

4. Show that the map $p:\mathbb{R}\to S^1$ given by the equation $p(x)=(\cos 2\pi x, \sin 2\pi x)$ is a covering map. [5]

5. Let $p: E\to B$ be a covering map. Show that if B is Hausdorff, regular, completely regular or locally compact Hausdorff, then so is E. [5]

6. Let $p: E\to B$ be a surjective covering map such that $p(e_0)=b_0$. If E is simply connected show that p is bijective. [5]

7. State and prove Brouwer fixed-point theorem for the disc. [5]

8. Show that if $h: S^1\to S^1$ is nullhomotopic, then h has a fixed point and h maps some point x to its antipode $-x$. [5]

9. Is there any continuous antipode-preserving map between S^2 and S^1? Justify your answer with acceptable reason. [5]

10. For each of the following spaces, the fundamental group is either trival, infinite cyclic or isomorphic to the fundamental group of figure eight. Determine for each space which of the three alternatives holds:
    a) The solid Torus, $B^2\times S^1$.
    b) The torus T with a point removed.
    c) $\mathbb{R}^3$ with nonnegative x,y and z axes deleted.
    d) $\mathbb{R}^2\setminus(\mathbb{R}_+\times 0)$.
    e) $\{x\in\mathbb{R}^2 \mid \|x\|\geq 1\}$. [5]

11. Compute the first simplicial homology group of $\mathbb{R}P^2$. [5]

12. Prove that the sequence of homology groups

$$\ldots\to H_n(A)\xrightarrow{i_*} H_n(B)\xrightarrow{j_*} H_n(C)\xrightarrow{\partial} H_{n-1}(A)\xrightarrow{i_*} H_{n-1}(B)\to\ldots$$

is exact. [5]

——— × ———